NuMBerS numerical methods for biosciences students

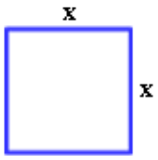
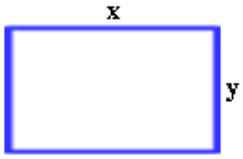
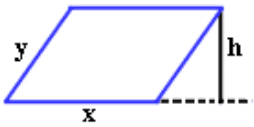
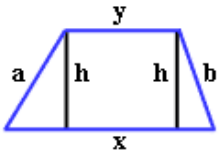
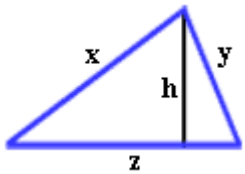
Basic numerical skills: AREAS, PERIMETERS AND VOLUMES RECIPE SHEET

1. Introduction

This sheet presents formulae for the perimeters and areas of some common two-dimensional shapes, and areas and volumes of some three-dimensional objects. Within the NuMBerS resource, there is also a spreadsheet that you can use to calculate the answers.

2. Two-dimensional shapes

2.1 Quadrilaterals (shapes with four straight sides) and triangles (shapes with three straight sides)

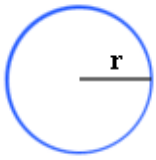
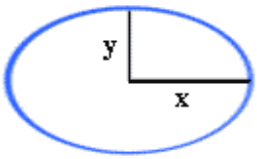
Shape		Perimeter	Area
Square *(all sides equal and opposite sides parallel, all angles right angles)*		4 **x**	$\mathbf{x}^2$
Rectangle *(opposite sides equal and parallel, all angles right angles)*		2 (**x** + **y**)	**xy**
Parallelogram *(opposite sides equal and parallel, corner angles not right angles)* (see also note below)		2 (**x** + **y**)	**xh**
Trapezium *(two opposite sides parallel but unequal)*		**x** + **y** + **a** + **b**	**h** (**y** + 0.5 (**x** - **y**))
Triangle *(three sides: special cases where all sides equal = **equilateral**; and two sides equal = **isosceles**)*		**x** + **y** + **z**	0.5 **hz**

Note: a parallelogram where all four sides are equal is called a **rhombus**

2.2 Circles and ellipses

A circle is the shape defined by the path of a point moving at a constant distance from another point. The constant distance is termed the 'radius' of the circle. An ellipse is oval in shape, which also comprises a single line which has no ends (ie it joins onto itself) and is defined by a major and minor axis. A circle is a special case of an ellipse, where the lengths of the two axes are identical.

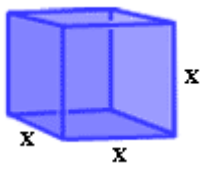
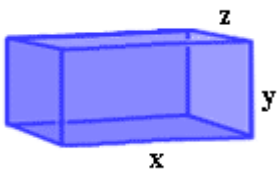
The quantity π used in the following formulae is the ratio between the perimeter of a circle and its diameter (the distance from one side of a circle to the other passing through the centre, or twice the value of the radius).

Shape		Perimeter	Area
Circle *(the distances from the centre to the edge of the circle is the same in all directions)*		$2\pi r$	πr^2
Ellipse *(a 'squashed' circle or oval, defined by the lengths of its major and minor axes)*		$2\pi\ (0.5\ .\ x^2\ .\ y^2)^{0.5}$ (see note below)	πxy

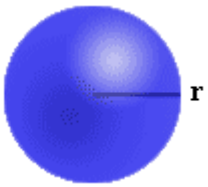
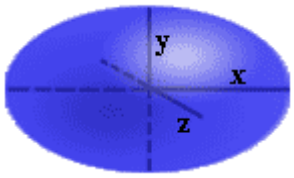
Note: the formula for the perimeter of an ellipse is an approximation

3. Three-dimensional objects

3.1 Cube and cuboids

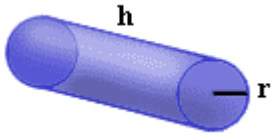
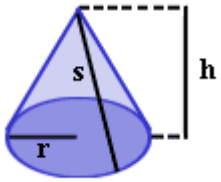
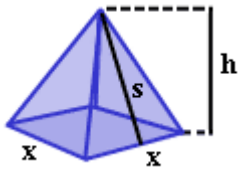
Shape		Area	Volume
Cube *(all faces are squares)*		$6\ x^2$	x^3
Cuboid *(all faces are rectangles)*		$2\ (xy + xz + yz)$	xyz

3.2 Sphere and ellipsoids

Shape		Area	Volume
Sphere *(any point on the surface is distance* **r** *from the centre)*		$4\,\pi r^2$	$4\,\pi r^3/3$
Ellipsoid *(distance between a point on the surface and the centre defined by the lengths of three axes)*		$4\,\pi\,((L+M+N)/3)^{1/p}$ where, $p = 1.6075$ $L = x^p y^p$ $M = x^p z^p$ $N = y^p z^p$ (see note below)	$4\,\pi xyz/3$

Note: the formula for the surface area of an ellipsoid is an approximation

3.3 Other objects

Shape		Area	Volume
Cylinder *(a prism with a circular cross-section and parallel long sides)*		$2\,\pi r\,(h + r)$ (see note below)	$h\,\pi r^2$ (see note below)
Cone *(a 3-D object with a circular base and sides that taper to a point or vertex above the centre of the base)*		$\pi r\,(s + r)$	$h\,\pi r^2/3$
Pyramid *(a 3-D object with a base – here a square – and triangular sides that meet at the vertex above the centre of the base)*		$x\,(2s + x)$	$h\,x^2/3$

Note: for any prism, the surface area is given by (the perimeter of the end times the length plus the areas of both ends), and the volume is the length times the area of the end.