Basic numerical skills: VARIABLES, PARAMETERS .... AND THE REST

1. Introduction

In statistics and other mathematical operations, you will use symbols to represent quantities in calculations and formulae. Although you can use whatever you like to represent things, there are some conventions that you will find in common use, and ways of showing particular properties and relationships. This document explains some of these, with the use of a series of tables of examples.

2. What are variables and parameters? (simple)

We use the term 'variable' to represent a quantity that can have a range of values. If we have a group of several people and measure the height of each person, height is a variable and can have a different value for each person.

Some variables can have any value within a range and are termed 'continuous' variables, whilst other variables can only have certain specified values or scores and are termed 'discontinuos' or 'step' variables. A numerical ranking is an example of a discontinuous variable.

Parameters (sometimes also called 'constants') are fixed values, at least within a given context. We may have a relationship between a person's height and their shoe size. This could be described in a word equation as:

$$\text{Shoe size} = \text{Height} \times R$$

Here, the quantity R has a single value. It has been derived by measuring the relationship between shoe size and height for several people. R is termed a parameter in the equation.

Constants are really special cases of parameters where they have been determined as universally-applicable quantities. For instance, in the famous equation linking mass and energy:

$$E = m.C^2$$

the symbol 'C' represents the velocity of light and is a constant value. The symbol 'm' represents mass, and is a variable, and determines the value of E, energy.

3. Introducing the notation used for variables (simple)

This section introduces the ways of representing a variable and the simple transformations (powers, roots and logarithms) that may be applied to it. In the following table, the variable is given the symbol '$\boldsymbol{a}$'.

a	A variable
a^2	The variable a squared (raised to the power 2)
a^x	The variable a raised to the power x (where x need not be a whole number)
a^{-1}	The reciprocal of a (that is one divided by a)
a^{-2}	The reciprocal of a-squared (that is one divided by a^2)
a^{-x}	The reciprocal of a raised to the power x
$a^{1/2}$ or $a^{0.5}$	The square root of a
$a^{1/x}$	The 'x^{th}' root of a
$\log(a)$ or $\log_{10}(a)$	The logarithm to the base 10 of a
$\ln(a)$	The logarithm to the base e of a (also called the 'natural' logarithm of a)
$\exp(a)$ or e^a	The quantity e raised to the power a (also called the exponent of a)

Logarithms and exponents are explained in other resources in the **NuMBerS** site.

4. Expressions (intermediate)

Expressions are ways of writing the interaction between different variables, or a variable and one or more parameters. In the relationship E = m.C^{2,}, 'm.C^{2}' is an expression that combines a variable (mass) and a parameter (the velocity of light).

Expressions should be written in a way that there is only one way to evaluate them. Where expressions are complex, brackets are used to indicate how the expression should be evaluated. The acronym BODMAS is used as a guide to evaluating expressions:

BRACKETS:	evaluate the contents of brackets first, working from the innermost (lowest in the hierarchy) to the outermost (highest in the hierarchy)
'ORDER':	evaluate powers
DIVISION AND MULTIPLICATION:	take priority over .....
ADDITION AND SUBTRACTION:	.... which are evaluated last

Conventionally, when the above priorities are followed, the expression is then evaluated from left to right. This is what computers and calculators do automatically, although you might find yourself selecting any part of a complex expression in order to start to simplify it. Very complex expressions may need to be broken up into separate sub-expressions that are evaluated separately.

The following table shows expressions including the variable 'a' and a parameter represented by the symbol 'b'. For clarity, multiplication is shown using a full-stop (period) as a separator, but this may not always be shown in expressions, especially if the multiplier is a number rather than a symbol. The multiplication sign '×' is not normally used in expressions, and lower-case x should never be used to represent this symbol. Division is written using '/' (backslash or solidus), and the division sign '÷' is not used.

a	A variable
$3.a$	The variable a multiplied by 3 (also written simply as '$3a$')
$b.a$	The variable a multiplied by b (also called the 'product' of a and b)
a/b	The variable a divided by b
$b.a^2$	The variable a-squared, then multiplied by b
$(b.a)^2$	The variable a multiplied by b, with the result squared (note the effect that the brackets have)
$(b.a^2)^3$	The variable a-squared multiplied by b, with the result raised to power three
$b.a + a^2$	The product of a and b, added to a-squared
$b.(a + a^2)$	The sum of a and a-squared, with the result multiplied by b (again, note the effects of the brackets)
$a + a^2/b$	The variable a-squared divided by b, added to a
$(a + a^2)/b$	The sum of a and a-squared, with the result divided by b (again, note the effects of the brackets)
$((a + a^2)/b)^{0.5}$	The square-root of the previous expression (inner brackets are evaluated first)

Operations on simple expressions are demonstrated in the '*Equations and how to solve them*' resource on the **NuMBerS** site.

5. Data series (intermediate)

Although you sometimes have only one value for a variable, you typically encounter several values. Where these values have some sort of order, they are usually stored in a series. Individual values in the series are denoted using a subscript, which can be thought of as a label or tag that identifies a particular position in the series. If the variable is one of several repeat measurements, so that there are several data series, the same subscripts will label the corresponding observations for each variable.

a_0	The first element or observation of a series of variable a (typically the starting value)
a_3	Element three of a series of variable a
a_i	Element 'i' of a series of variable a
$a_i . x_i$	Element 'i' of a series of variable a, multiplied by the corresponding value for the variable x (that is the values of a and x come from the same observation or sample)
$a_{0..i}$	The series of elements a_0 to a_i
a_{i+1}	The next element in the series of variable a above a_i
a_{i-1}	The next element in the series of variable a below a_i
a_3^2	The square of element three of a series of variable a
a_3^{-1}	The reciprocal of element three of a series of variable a
$a_3^{0.5}$	The square root of element three of a series of variable a
a_i^x	The x^{th} power of the i^{th} element of a series of variable a
$b.a_i^x$	The x^{th} power of the i^{th} element of a series of variable a, then multiplied by b
$(b.a_i)^x$	The i^{th} element of a series of variable a multiplied by b, with the result raised to power x

Sometimes it is useful to order measurements of a variable in a two-way classification, which can be visualized as rows and columns in a table. The elements are then identified by a double subscript like $a_{i,j}$.

Particular notation is used in statistical calculations, where typically various totals of squares and products are used to derive values that contribute to the calculation of statistics. Subscripts are normally used in statistics to denote something other than position in a series. The upper-case Greek letter sigma (written Σ) is used to denote a sum (ie all values added together). The following table includes some expressions that you are likely to encounter (variables in these examples are x and y).

Σx	The sum of all the values for the variable x
Σx^2	The sum of the squares of all of the values for the variable x
$(\Sigma x)^2$	The square of the sum of all the values for the variable x
Σxy	The sum of the products of all values of the variable x and the corresponding values of the variable y